Quantum Moduli Spaces and TQFT

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Abstracts

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Dorin Cheptea

Finite-type Invariants of 3-dimensional Manifolds, TQFT, and Beyond

Abstract: This talk is a quick overview of several results. The universal finite-type invariant of 3-manifolds of Le, Murakami and Ohtsuki has a functorial property and fits in the framework of an anomaly-free TQFT, leading to infinite-dimensional representations of the Torelli group. Moreover, the (combinatorial) composition of chord disgrams that corresponds to the (topological) composition of cobordisms can be expressed in purely combinatorial terms. This opens several directions for computing the functor.

Alexander B. Goncharov

Higher Teichmüller Theory

Abstract: Given a topological surface S and a split simple Lie group G, we define a pair moduli space related to S and G. We show that they admit positive atlases. The sets of their real and tropical points generalize the Teichmüller space and the lamination space for S. We quantize the higher Teichmüller spaces.

This is a joint work with V.V. Fock.

Lars Hesselholt

Combinations of Polytopes and Algebraic K-Theory

Abstract: The regular cyclic polytope of dimension $2d$ with m vertices is defined to be the convex hull of m evenly spaced points on the trigonometric moment curve

$$t \mapsto (\cos t, \sin t, \cos 2t, \sin 2t, \ldots, \cos dt, \sin dt).$$

The regular cyclic polytopes are higher dimensional generalizations of the regular polygons, and their combinatorial structure is completely understood. In this talk, I will introduce a new class of polytopes that generalize the class of cyclic polytopes. The combinatorial structure of these new polytopes is presently understood only in dimensions 2 and 4. I will also formulate a conjecture about these polytopes and explain how this conjecture, if confirmed, leads to a complete evaluation of the algebraic K-theory of planar cuspical curves.

Franz Kamber

Equivariant Transversal Index Theory

Abstract: The equivariant analytic index of a G-equivariant differential operator on a compact G-manifold M, which is transversally elliptic with respect to the group action on M, is a distribution valued virtual character (Atiyah). The index multiplicities $\mathrm{Ind}_\rho(D)$ of type ρ are finite, but until now very little was known about the problem of expressing them in terms of the topological and geometric data involved. In particular, this is true for the *invariant* index $\mathrm{IndG}(D)$ associated to the trivial representation. Our main result is a reduction formula for the invariant index in terms of the strata of the group action. The main term is a formula of Atiyah-Singer type over the stratum of principal orbits, with additional terms involving η-invariants being contributed by the behaviour of D near the singular strata.

Rinat Kashaev

A Quantization of the Moduli Space of Flat $PSL(2,R)$-Connections on Punctured Surfaces

Abstract: Penner's decorated Teichmüller space of a punctured surface can be generalized to a decorated moduli space of flat irreducible $PSL(2,R)$-connections with parabolicity conditions around punctures. This space, similarly to the decorated Teichmuller space, admits a parametrization which is well suited for quantization. The solution of the pentanonal equation of Woronowicz and Zakrzewski can be used for quantization. It restricts the deformation parameter to belong to only an infinite discrete set given by certain roots of unity. The main outcome of the constructed quantum theory is an infinite-dimensional unitary projective representation of the surface mapping class group.

Gregor Masbaum

Integral TQFT and Perturbative Expansion

Abstract: We study the mapping class group representations associated to the SO(3) TQFT at an odd prime. The main result of this talk is that when restricted to the Torelli group, these representations stabilize (in a sense to be explained) as the order of the root of unity goes to infinity. Here it is crucial to use the integral bases we found in previous joint work with Pat Gilmer. This gives in particular a skein-theoretical construction of Ohtsuki's power series invariant of homology spheres.

Robert Penner

The Punctured Solenoid

Abstract: The punctured solenoid is the inverse limit of all finite unbranched covers of a punctured surface of negative Euler characteristic, which is the punctured analogue of the universal hyperbolic solenoid introduced by Sullivan to study various problems in geometry and dynamics. In each case, the Teichmüller theory can be developed along classical lines, and the solenoid is seen to be universal in the sense that its Teichmüller space contains densely those of all surfaces of finite type. For the punctured solenoid, there is a decorated Teichmüller space with global coordinates, a computable mapping class group action, an invariant canonical cell decomposition and an invariant two-form as in the case of finite-type punctured surfaces. We shall discussf this ongoing work, which is joint with Saric and Bonnot/Saric.

Michael Polyak

Real Enumerative Geometry and Finite Type Invariants

Abstract: Complex enumerative geometry deals with counting algebraic-geometric objects satisfying certain restrictions, e.g., counting algebraic curves of a fixed degree which satisfy given passage/tangency conditions. I will discuss various real counterparts of such problems, explain their relation to the theory of finite type invariants and propose a general setting to produce such invariants using maps of configuration spaces.

Nicolai Reshetikhin.

Integrability and Invariants of Knots

Abstract: The talk will be focused on some aspects of invariants of knots. Jones polynomial and related invariants will be key examples. First part will be a survey of how Chern-Simons invariants, moduli spaces, and quantum groups are related to the Jones polynomial. Then the focus will shift to Khovanov homology. Finally, few words will be said about Chern-Simons versus topological string duality. This last subject made a substantial progress recently moving from vague ideas originated in string theory to a precise mathematical statement known as Donaldson-Thomas Gromov-Witten duality.

Sergei Shadrin

Tautological Equations in dGBV Algebras

Abstract: I'll dicuss two different classes of solutions of the WDVV equation. The first one is related to the intersection theory of the moduli space of curves (Gromov-Witten invariants), the second one comes from deformation theory (Barannikov-Kontsevich construction). Both classes of solutions admit natural genus expansion and the goal of the talk is to explain how one can compare these genus expansions. The talk is based on my joint papers with A.Losev and I.Shneiberg math.QA/0506039, math.QA/0507106, and math.QA/0507107.

Catharina Stroppel

Khovanov Cohomology as a "Shadow" of Representation Theory and Geometry

Abstract: In this talk we give a reprentation theoretic explanation of Khovanov cohomology and explain the relationship with the cohomology of Springer fibres. As a result we deduce that Khovanov cohomology emerges naturally from representation theory of Lie algebras. On the other hand we can give a purely diagrammatic description of the category of representations in question by extending Khovanov's original definition. Finally we want to explain the connection to (the cohomology of) Springer fibres and coordinates rings of nilpotent orbits and demonstrate how they appear naturally in both pictures.

Dylan Thurston

A Combinatorial Definition of Heegaard-Floer homology

Abstract: We present work of Manolescu, Ozsvath, Sarkar, and Wang, giving a completely combinatorial presentation of Heegaard-Floer homology for knots, links, and 3-manifolds.

Dylan Thurston

Cluster Algebras and the Geometry of Surfaces

Abstract: Cluster algebras are a new way of thinking about coordinates on Teich-müller space. They come in three varieties: tropical (controlling the topology of simple curves on the surface), algebraic (controlling Teichmüller space), and quantum. The technology of cluster algebras suggests generalizations, including a new way of dealing with surfaces with boundary or closed surfaces with no punctures.

Alexandr Usnich

Quantization of Birational Symplectomorphisms of $\mathbb{C}P^2$

Abstract: The group of birational automorphisms of $\mathbb{C}P^2$ preserving logarithmic poisson bracket admits a morphism to the group of piecewise linear automorphisms of $\mathbb{Z}^2$, known as the Thompson group T. This morphism helps to clarify the structure of both groups and to introduce some simple birational automorphisms, playing role of mutations in cluster algebras. We make conjecture about the structure of the group, generated by such mutations, implying that this group also acts on non-commutative P^2's. This picture is coherent with the quantization of Teichmüller spaces by Fock and Goncharov.

Nathalie Wahl

Homological Stability and a Coloring Lemma

Abstract: (joint work with A. Hatcher) We use a coloring lemma to prove homological stability for the mapping class groups of 3-manifolds under the operation which glues extra handles ($S^1 \times S^2$'s) to the manifold. In the case where the manifold is a connected sum of $S^1 \times S^2$'s, this provides a new simpler proof of homological stability for the automorphism groups of free groups.

Chris Woodward

Lagrangian Correspondences and Floer Field Theory

Abstract: This is joint work with K. Wehrheim. We associate to any Lagrangian correspondence a functor between Donaldson-Fukaya categories, and prove that composition is well-behaved. Applied to moduli spaces of flat $\mathrm{SU}(r)$-bundles with fixed holonomies around codimension two submanifolds, this gives a 2+1 dimensional category-valued field theory, roughly speaking, with the same exact triangle as Khovanov-Rozansky.

Milen Yakimov

Poisson Structures on Moduli Spaces of Flat Connections on Riemann Surfaces

Abstract: We will discuss the geometry of a Poisson structure on a rational quotient with respect to the action of a complex simple algebraic group G on the representation variety associated to a surface with punctures and the group G.

This is joint work with N. Reshetikhin and R. Kashaev.